Standard Form

We use standard form whenever a number is too small or too large to write easily in the usual way - this is used all the time in economics, science & business.

1. Write £2,300,000,000 in standard form. (This is the total yearly income of the UK)
2. Express 6.7 x 10⁷ as a normal number. (This is the UK’s population)
3. Calculate the GDP per capita (which is ’average yearly income per person’) in the UK, giving your answer in standard form.
4. If £1 = 1.27$, how much is this in $?
5. Express 6,000,000,000,000,000,000,000,000 kg in standard form. (This is the mass of the Earth, M)
6. If the radius of the Earth is r = 6.4 x 10⁶m, calculate r². (: multiply in standard form!)
7. If a falling object accelerates at

calculate using your standard form values of M and .

1. If a star is 1,000 Light-Years away (1 Light-Year = m), calculate:

a) the distance in meters,

b) the distance in km,

c) the time taken for light from the star (speed = meters per second) to reach the Earth in seconds, using your answer from part (a)

d) If 1 year = , how many years does it take for the light to reach the Earth?

1. The table below shows some data:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Planet** | **Radius** | **Mass** | **Volume** | **Time taken to orbit** | **Mean orbital distance** |
| **Earth** | 6370km | 6.0⨉10²⁴kg |  |  | 1.0AU |
| **Jupiter** | 69,900km | 1.9⨉10²⁷kg |  | 3.7⨉10⁸s | 5.2AU |
| **Uranus** | 25,400km | 8.7⨉10²⁵kg |  | 2.6⨉10⁹s | 20AU |
| **Pluto** | 1190km | 1.3⨉10²²kg |  | 7.8⨉10⁹s | 49AU |

1. Convert the radius of each planet to metres, and give the answers in standard form.
2. Use the formula for the volume of a sphere, (r = radius) to calculate the volumes of each planet (in m³). Fill in the **Volumes** column with volumes in standard form.
3. Calculate the number of seconds in 1 year, and fill in the **Time taken to orbit** column.
4. How many Earth years does it take Jupiter, Uranus and Pluto to orbit the Sun?
5. If 1AU = 1.5⨉10⁸km, calculate the mean orbital distance from the Sun for each planet in standard form
6. Assuming that the planets orbit in circles around the Sun, find the circumference () of the orbit of each planet. Give your answer in standard form.
7. Why is the value given the ‘mean’ orbital distance? What is wrong with our assumption in part (f)?

Geometry of Circles & Spheres

Circles and spheres are the simplest 2D and 3D shapes (since they are the same in all directions), and are therefore some of the most mathematically important.

Circle: area = , perimeter =

Sphere: volume = , surface area =

1. If a circle has a radius of 10cm, what is

a) its perimeter, and

b) its area?

1. If a sphere has a diameter of 18cm, what is

a) its volume

b) its surface area?

c) What is the ratio between volume and surface area?

1. If Pluto has a radius of m and Venus has a radius of m, calculate:

a) the volume & surface area of each planet.

b) What is the ratio of volumes ()?

c) What is the ratio of surface areas?

d) What is the ratio of radii?

1. What is the volume of a cylinder with a 0.5m radius and a depth of 3cm?

b) If this is a model of a bike tyre where the metal frame of the wheel has a radius of 0.45m, what is the total volume of the tyre? (HARDER. HINT: Try and draw the shape, and try subtracting one cylinder volume from another cylinder volume.)

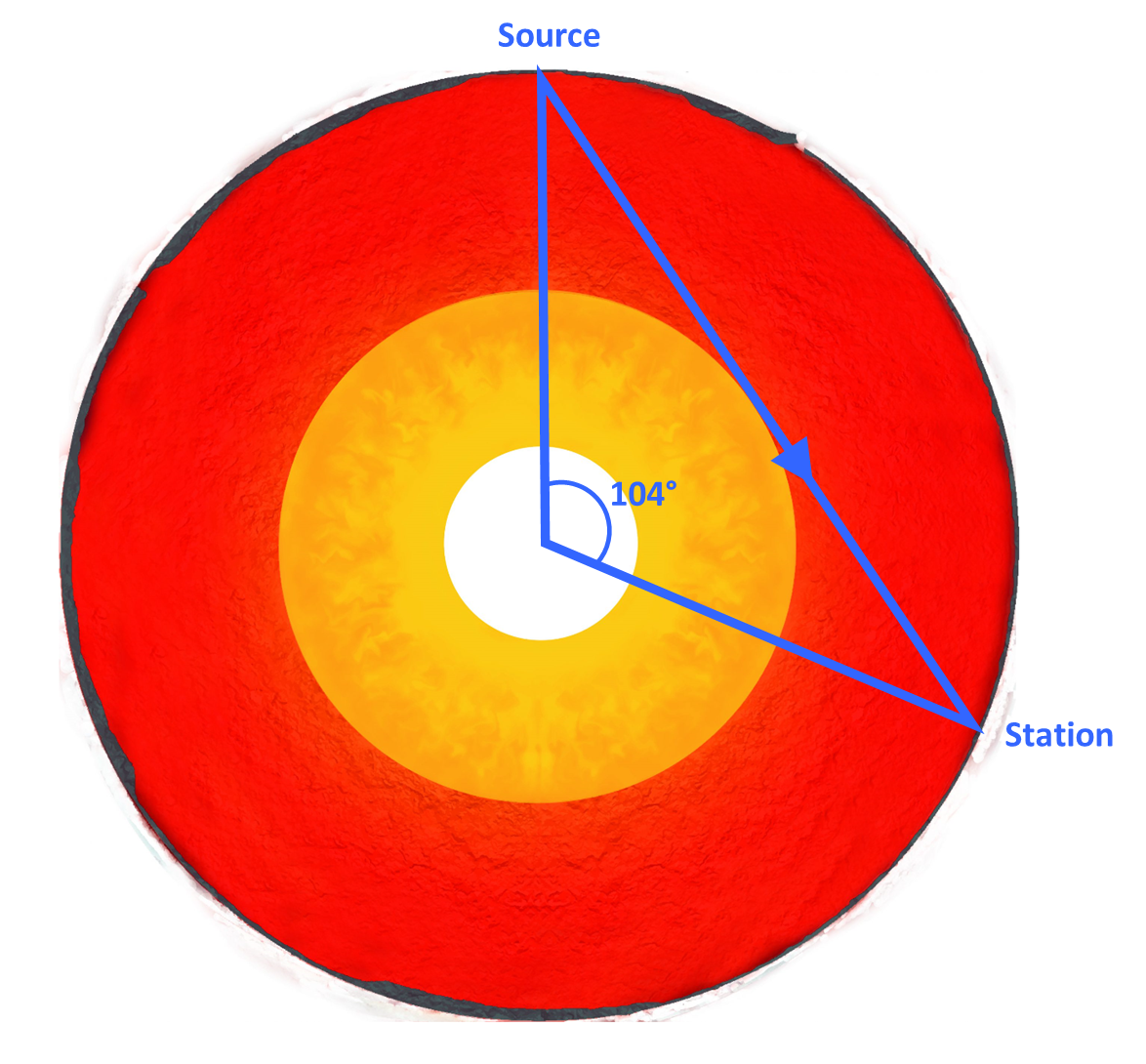
1. What is an equation for the total surface area of a half-sphere? (HARDER. HINT: Draw a half-sphere and see what it looks like, and then add up the surface areas of both faces, one curved & one flat)
2. Earthquake energy travels through the Earth as ‘seismic waves’.

Seismic waves can be detected by seismometer stations at many different locations around the Earth due to an earthquake in a particular location.

However, none of these seismic waves travel more than 104° (measured from the centre of the Earth) round the circumference of the Earth.

This is due to the size of the core and the fact that seismic waves can’t travel through the core easily.

Use this fact to calculate the radius of the core of the Earth, given that the entire radius of the Earth is 6370km.



Mass, Density and Volume

Mass (M) is the amount of ‘stuff’ you have. Volume (V) is the amount of space that ‘stuff’ takes up. Density (D) is the ratio of mass to volume, .

1. a) If a brick has the dimensions 20cm x 10cm x 10cm, calculate its volume. If this brick has a mass of 3000g, calculate its density.

b) What are the units of density in this case?

1. A sphere of osmium (the highest-density metal!) has a radius of and a density of . What is the mass of this small ball of metal in kg?
2. a) Oil has a density of . Water has a density of . What is the overall mass of oil + water?

b) You may have heard that oil and water don’t mix in a glass: which one (oil or water) would settle on top, and which on the bottom?

1. of sand at the surface of Earth has a mass of. of sandstone formed 500m underground has a mass of . Calculate the densities of each. Why do you think one is denser than the other?
2. The pressure of air is related to the density of air by

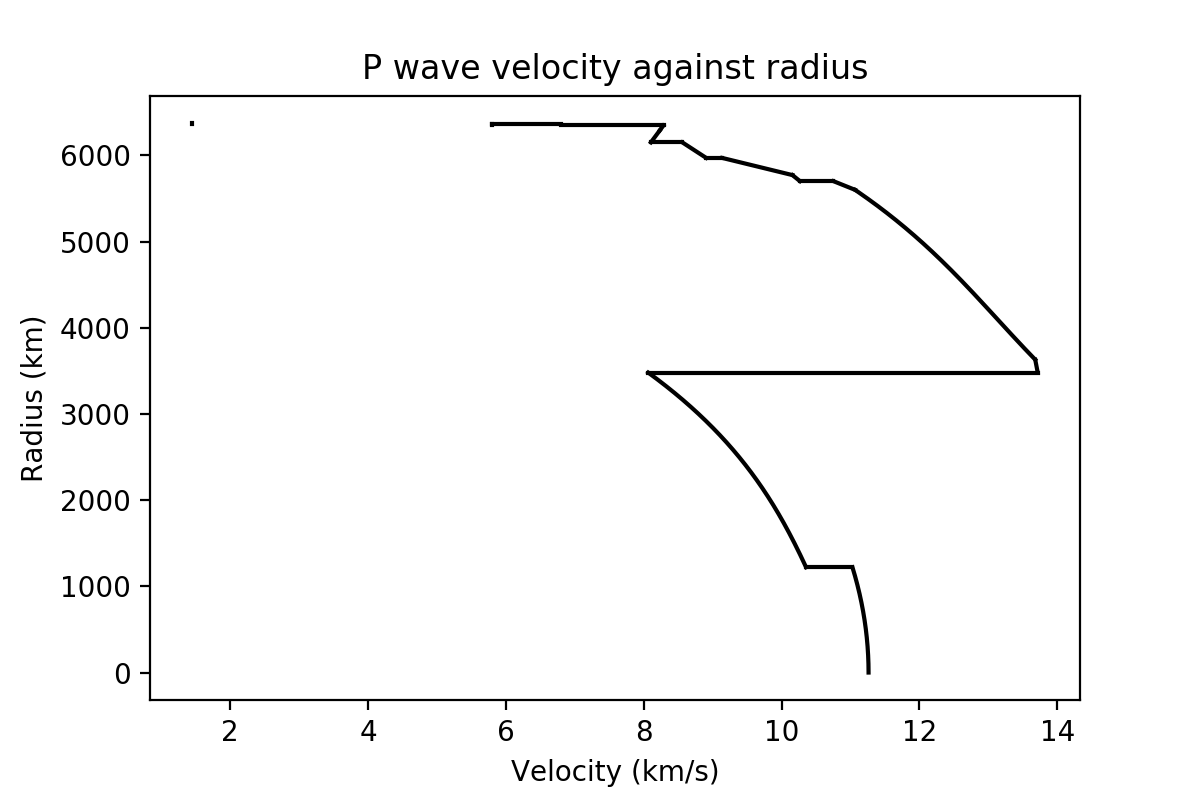
At an atmospheric pressure Pa, calculate the density of the air when temperature is and when . Describe, in your own words, the mathematical relationship between Temperature and Density.

Percentages and Proportions

1. There areadults in the UK who are working age. Of these, 17 million have the numeracy skills expected at KS2.
2. What percentage is this?
3. 42% of working-age adults are University graduates, 21% have only A-levels and 20% have only GCSEs. Calculate the number of adults in each category.
4. Estimate the ratio of graduates:A-levels:GCSEs.
5. 10% of the world’s Earthquakes happen in Japan each year.
6. If 1500 Earthquakes happen here, how many happen in the world in total?
7. Around 90% of Earthquakes happen around the “Pacific Ring of Fire” (at the edges of the Pacific Ocean). How many Earthquakes is this per year?
8. How many Earthquakes happen in Japan per day, and how many happen in the Pacific Ring of Fire per day?
9. Around 25 Earthquakes are felt in the UK per year - how many days is it between each Earthquake, on average?
10. a) Due to COVID-19, joblessness claims rose by 70% to around people. What was the number of people claiming joblessness beforehand?

b) 11% of the population are now claiming unemployment benefits in Blackpool compared to 7% before Coronavirus. There are people in Blackpool. How large is the increase in unemployed people in Blackpool due to Coronavirus?

Graph skills



1. This is a graph of some Earthquake waves (“P wave”) velocities through the Earth.
2. Whereabouts in the Earth do you think the “0km” and “6300km” radii are found?
3. How many distinct regions of the Earth can you make out?
4. What do the horizontal sections, such as at around 3400km and 1200km, represent?
5. At what radius is i) the highest Earthquake wave speed and ii) the lowest Earthquake wave speed?
6. Describe, in a short paragraph, how you might explain the trends shown on this graph? Given that Earthquake wave velocities move surprisingly slowly through liquid, at what depths in the Earth is there likely to be a liquid layer?

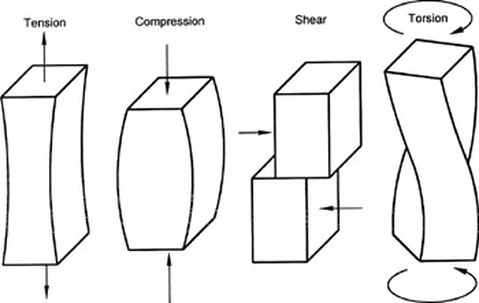
Algebra

1. Earthquake waves in the Earth come in two varieties: “P waves” and “S waves”. These have different speeds when travelling through the Earth, calculated by:

S = resistance to shearing, units: Pascals

C = resistance to compression, units: Pascals

D = density of material, units: kilograms per cubic meter ()



1. Write down expressions for and .
2. Rearrange equation (i) to find D, the density.
3. Rearrange equation (ii) to find S, the resistance to shearing
4. If ,and , calculate and . Which is faster?
5. Find an expression for .
6. A liquid has a shear resistance of . (You’ll know this if you’ve ever tried to shear water - the concept doesn’t make any sense!) Write down equations for and in this case (hint: isn’t really an algebraic equation anymore, but just a number.)